Extended Abstract:
„Unsplittable Transshipments[1]“

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Keywords: Unsplittable transshipment, wholesale logistics, capacity-aware routing, B2B distribution, network congestion, route planning, supply chain optimization

In wholesale logistics, the ability to move goods from multiple warehouses to a network of retail outlets is central to operational efficiency. Each shipment, from a specific warehouse to a particular store, must travel as one indivisible unit. Pallets, containers, and truckloads cannot be split and rerouted in fragments without introducing costs, inefficiencies, or regulatory complications. At the same time, the transport network has strict limits: roads can only handle so much traffic, hubs have limited throughput, and trucks must not exceed their loading capacities.

The central question is: **How can goods be routed through such a network so that each shipment follows a single path, no road or hub is overloaded, and every store gets what it needs?**

We study the abstraction of this problem, which we call **unsplittable transshipments (UT)**. This mathematical framework provides a rigorous model to encode the aforementioned logistical constraints.

Main Findings

Our research is foundational and studies the abstract version of the UT problem. It shows that the UT problem is computationally challenging in the general case. In fact, it is **strongly NP-hard**, which means exact solutions cannot be expected in reasonable time for all possible network configurations. This is an important baseline result: it tells us why wholesale distribution under indivisibility constraints is complex and why careful algorithm design is needed.

Despite this theoretical hardness, we achieve the following results:

1. **Bounded capacity violations**
    - In situations where some overloading is unavoidable, the algorithm ensures that no connection exceeds its capacity by more than the size of a single demand.

[1] *Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany´s Excellence Strategy – The Berlin Mathematics Research Center MATH+ (EXC-2046/1, project ID: 390685689)*

- We also show that this congestion is as low as mathematically possible in general networks, assuming the total demand is feasible within the network's overall capacity.
- For wholesale operators, this means overloads remain within predictable, controllable limits.
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2. **Scalability**
   - The algorithm runs in polynomial time and scales effectively with network size. This makes it suitable for large distribution networks and for integration into digital planning tools.
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3. **Efficient two-round planning**
   - Many wholesale operations are organized in waves, rounds, or time windows. A key question is: how many rounds are needed to deliver all goods unsplittably without exceeding capacities?
   - We found that if each round must itself be a valid unsplittable transshipment (i.e., each shipment follows a single route, and no road or hub is over capacity), then **two rounds are always enough**, provided the demands can be met by the total network capacity.
   - This gives the following operational insight: no matter how complex the network, goods can always be delivered in at most two phases. For wholesalers, this result could provide a concrete scheduling framework that balances indivisible shipments with infrastructure constraints.

Applications for Wholesale Logistics

These findings directly support wholesale logistics in several areas:

- **Dispatch planning under indivisible loads**
  Ensure that shipments like pallets or truckloads are routed as whole units, while still respecting network capacities.
- **Digital decision systems and twins**
  Integrate algorithms into logistics control towers and planning tools, to design routing options quickly and without trial-and-error.
- **Structured batch planning**
  Organize deliveries into one or two predictable rounds, simplifying timetables and resource allocation while maintaining efficiency.

Conclusion

We analyze the **unsplittable transshipment problem**, an abstract mathematical model that captures the realities of wholesale logistics more faithfully than some other traditional flow models. While the problem is theoretically difficult, we provide some foundational results with immediate relevance: an efficient algorithm with mathematically tight congestion guarantees, predictable and bounded overloads, and a demonstration that all goods can be routed in just two rounds under realistic operational constraints.

For wholesale distributors, service providers, and technology partners, these results offer a reliable framework for designing distribution networks, managing bottlenecks, and making B2B logistics both more predictable and more efficient.